Index Futures Mispricing and Ambiguity*

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Abstract

The present paper empirically unveils the effect of having multiple priors on mispricing in the market where mean-variance optimization and the Bayesian approach do not have any say. We show that the level of mispricing in S&P500 E-Mini futures contracts is also linked to the degree of prevailing market ambiguity. Crucial findings are in order. First, our study unearths how different levels of Knightian uncertainty impact the direction and level of mispricing in US futures markets. Second, profound analysis reveals an asymmetric outlook for episodes of market euphoria and unrest. Third, we identify the primary channels through which the ambiguity permeates the market. Findings are robust to different ambiguity measurement techniques. Extant literature on marred prospects and market implications rests heavily on experimental data. This study expands thin ambiguity literature utilizing real market data.

PS: We are now focusing on the main drivers of futures markets. Hazelkorn et al. (2023) show that liquidity demands from different market actors shape futures-cash basis. In a similar fashion, we are currently compiling data from the Commodity Futures Trading Commission to work on the supply & demand dynamics and cross-market activities of different clientele.

JEL classification: G12; G13; G15; G40; D81

Keywords: Behavioral Finance; Ambiguity; Asset Prices; Index Futures; High-frequency Data

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1. Introduction

In its technical design, futures contract prices converge to spot prices at expiration dates. As a natural outcome, basis movements are supposed to have a negative autocorrelation and given a non-stochastic interest rate term structure and dividend payout, they are completely deterministic according to the cost of carry model (Miller et al. (1994)). However, voluminous literature documents deviations of index futures market prices from theoretically generated cost-of-carry prices (Cornell (1985), Cornell and French (1983), Dwyer et al. (1996), Figlewski (1984), Kawaller et al. (1987), MacKinlay and Ramaswamy (1988), Tu et al. (2016) among many others.) Settled wisdom says that any price deviation from no-arbitrage boundaries will attract arbitrageurs for price correction. However, the mispricing may remain unexploited due to lack of arbitrage capital (Rubinstein (1987), Shleifer and Vishny (1997)), increased uncertainty for dividends with time to maturity (MacKinlay and Ramaswamy (1988), Yadav and Pope (1994)), short-selling restrictions (Brenner et al. (1989)), varying liquidity conditions (Hirshleifer (2001), Kamara and Miller (1995)), limitation for efficacious judgements due to staleness in spot index (Brenner et al. (1989), Richie et al. (2008)), and expected volatility (Tu et al. (2016)).

To insure their existing investment positions, market participants use various financial instruments one of which is the index futures contracts. However, knitting the analyses based only on the forces in index futures markets will be prosaic as today's complex market structure entails extricating different dynamics to grasp the picture comprehensively. As a matter of fact, Baltussen et al. (2021) recently documented that market momentum in index futures is also linked to hedging demands emanating from net gamma exposures of different clientele like option market makers, portfolio insuring institutional investors and leveraged ETFs. Our study further examines if these markets forces are linked the prevailing market ambiguity behind the scene by specifically focusing on the mispricing in index futures.

A century ago, Knight (1921) addressed the issue of "unmeasurable uncertainty" in which no probability measures are assigned for the subsets of a potential outcome space. In the same spirit, Keynes (1936) pointed out that majority of our decisions are steered by animal spirits rather than pure mathematical expectations. Savage (1954) claims that agents circumvent uncertainty by leaning on their subjective priors during utility maximization. However, confronted with a choice between risk and uncertainty, decision makers eschew from vague outcomes and prefer risk to uncertainty (Ellsberg (1961), Epstein and Wang (1994), Fellner (1961) among many others).[1]

One branch of the literature has extensively reported that market ambiguity shapes investor behaviours. It is without doubt that obscure future states is a source of unrest for market participants. Mukerji and Tallon (2001), Easley and O'Hara (2009) and Jiang and Zhu (2017) report that ambiguity aversion may hinder certain investors from trading certain financial assets due to blurred expected return levels.[2] Leading to non-participation in these assets, ambiguity aversion adversely affects risk sharing in the market. Similarly, Trojani and Vanini (2004) show that aversion to ambiguity reduces equity market participation[3]. Consistently, models that incorporate ambiguity imply less frequent portfolio reshuffling compared to classical mean-variance optimization and Bayesian approach which are silent to ambiguity (Garlappi et al. (2007))[4]. Agents also demand higher risk premium when the prospects are suffused with vagueness (Camerer (1995), Hirshleifer (2001), Trojani and Vanini (2004))[5].

Persistence of index futures mispricing is also connected to trading practices by the arbitrageurs. Literature (Dwyer et al. (1996), Klemkosky and Lee (1991), Shleifer and Vishny (1997)) and practitioner evidence show that arbitrage activities are mostly undertaken by Exchange members and institutional / professional investors due mainly to advantageous cost tariffs, capital requirements, well equipped teams, and continuous monitoring. To put it differently, retail investor involvement in arbitrage activities is on negligible levels. That said, reaping potential arbitrage profits in financial markets is not a bird in the hand. As opposed to riskless profit perception, executing arbitrage activity is mostly risky and not completely mechanical. Concerned with the return performance of entrusted capital, arbitrageurs may be forced to liquidate their positions dur-

[1] Heath and Tversky (1991) expand the discussions on ambiguity aversion and cast light on judgmental probabilities to indicate that people may opt for ambiguous outcomes when they have a feeling of competence or prior knowledge on the subject. Digging for further clarification, Fox and Tversky (1995) bring up "comparative ignorance hypothesis" to assert that ambiguity aversion is extant when agents make joint assessment of cloudy and clear prospects and this aversion may wane if these prospects are evaluated independently. See also Trautmann et al. (2011) for preference reversals and the way ambiguity aversion is measured.

[2] Easley and O'Hara (2009) offer making regulatory amendments (especially for the worst-case /extreme events) to solve this participation problem. See also Dimmock et al. (2016) for an empirical study on ambiguity and market participation.

[3] In their empirical study, Antoniou et al. (2015) similarly document negative relation between ambiguity and equity fund flows.

[4] Similar to Garlappi et al. (2007), Bossaerts et al. (2010) report relatively more balanced portfolio formation by ambiguity averse agents. In the same vein, Illeditsch (2011) reports portfolio inertia for ambiguity averse investors.

[5] Leippold et al. (2008) incorporate learning and ambiguity to explain high equity premia. Easley and O'Hara (2009) similarly report higher level of returns for stocks that are not much preferred by ambiguity averse investors.

ing persistent noise trader shocks and shun price correction especially when asset mispricing is acute (Hirshleifer (2001), Shleifer and Vishny (1997))[6]. Chasing the liquidity during extreme market conditions is also a shared trading behavior among high frequency traders which is triggered mainly by risk management concerns for over-accumulated positions (Brogaard et al. (2018)). Our motivation to figure out the relation between ambiguity and mispricing in index futures is also grounded on arbitrageurs' sensitivity to uncertainty. Arbitrageurs are clustered in bond markets due to their relative confidence in fundamental valuation in contrast to stock markets for which it is harder to assess fundamental values because of stochastic cash flows (dividend amounts) and their timings (Shleifer and Vishny (1997)). As the authors point out further, stock market uncertainty dissipates slowly and mispricing in stock markets can remain intact for a long-time (especially during volatile periods).[7] This makes the market less attractive to arbitrageurs who are generally concerned with short-term return performances. Popularity of equities decrease in portfolio formation practices as well; investors reduce the weight of equities and slant their preference towards riskless assets in an ambiguous environment.[8]

At first glance, the closest study to ours seems to be Tu et al. (2016) in which the authors link nonvanishing index futures mispricing to expected volatility levels represented by 30-day VIX numbers. To the extent implied volatility proxies for risk, arbitrageurs' ramping risk aversion and associated higher compensation expectancy dissuade them from price correction. However, their study can be associated with the literature that elaborates on market participation costs and risk aversion simply put.[9] Also strikingly, expected volatility and expected ambiguity can already shape investor decisions in different ways. For instance, higher levels of the expected volatility is associated with later stock option exercises whereas higher levels of the expected ambiguity backdates these option exercises (Izhakian and Yermack (2017)). Hence, pillars of the present study starkly differ from the aforementioned paper.

[6]See also Kyle and Xiong (2001) to behold how wealth effect may force rational convergence traders to abstain from exploiting deviations from fundamentals during severe market conditions.

[7]Epstein and Schneider (2008) and Illeditsch (2011) present how ambiguous signals worsen excess price volatilities compared to a Bayesian framework. Refer also to Caskey (2009) to see how persistent mispricing may be linked to information packages and ambiguity averse investors.

[8]See Maenhout (2004) for a detailed discussion.

[9]See Paiella (2007) for further discussion on participation costs.

2. Data, Model and Methodology

2.1. Data

Our data spans from October 1997 to December 2021 with daily frequency.[10] This period covers more than 24 years within which the markets had experienced numerous different cases; dot.com bubble and burst, 11/9 terrorist attacks, global financial crises, plethora of funds via quantitative easing in the follow-up period, European debt crises, negative interest rates, Brexit, Covid-pandemic and many other ups and downs in global markets. Markets also had gone through enormous technological transformations where algorithms had had considerable impact in trading practices especially in developed markets. Versatility in market conditions fortifies the implications and generalizability of our findings.

We extract S&P500 daily spot index levels along with accompanying composite dividend yields from Refinitiv Database. The risk free rate in cost-of-carry model comes from FED three-month treasury bill yields.[11] E-mini futures are the front-month contracts where the trading volume is largest. Daily contract settlement prices and VIX numbers are similarly taken from Refinitiv. Similar to our model construction rationale, Kostopoulos et al. (2022) uses VSTOXX (equivalent of VIX for Europe) for volatility measure and volatility of VSTOXX (V-VSTOXX) as the ambiguity measure in their analysis of investor behaviors amid market ambiguity and they strikingly report that ambiguity measure (not the volatility) is statistically significant in explaining the risk-taking behaviors of investors.

Futures mispricing data series (MP_t) is constructed by first deducting cost-of-carry theoretical price from the market price and expressing it as the fraction of spot index level at that date as formulated in Eq. (1).

$$F^*_{t,T} = S_t e^{(r-d)(T-t)} \quad \text{and} \quad MP_t = \frac{F_{t,T} - F^*_{t,T}}{S_t} \tag{1}$$

That said, the level of mispricing that will presumably urge arbitragers to step in and drag the prices to no-arbitrage region depends on the cost structures in the market. We formulate boundary

[10] E-mini futures contracts were first launched for trading in September 1997

[11] Yields are available at the following website: https://www.federalreserve.gov/datadownload/Choose.aspx?rel=H15

violations as in the following equation.

$$Bvio_t = max(|MP_t| - C_t, 0) \tag{2}$$

where C_t represents the cost levels associated with arbitrage practices. Costs mainly include the two-way commissions, slippage costs and bid-ask spreads. Those costs are expressed to be varying for Exchange members and professional investors. Dwyer et al. (1996) assume the cost levels as 0.25% of index value for NYSE Members and 0.38% for institutional investors. Extrapolated cost levels range between 0.0%-0.2% of spot index level in Richie et al. (2008); which is similar to Tu et al. (2016). In our main analysis, we too use 0.2% as the threshold parameter.

2.2. Model and Methodology

2.2.1. Ambiguity Measurement

Empirical counterparts of ambiguity measure are not ample. Cao et al. (2005) use variation in the mean levels as a proxy for uncertainty. Likewise, Garlappi et al. (2007) set confidence intervals around expected return levels and link portfolio choices to the precision in mean estimation. Shortly later, analyst forecast dispersion offered by Anderson et al. (2009) has become a popular ambiguity proxy in the literature. Antoniou et al. (2015) similarly use this dispersion as the surrogate measure for ambiguity while studying stock market participation and Lee et al. (2019) prefer this measure to explore uncertainty and cross-sectional expected stock returns.[12] In quantifying the degree of ambiguity, this study employs a recently introduced methodology that is present both in Izhakian and Yermack (2017) and Brenner and Izhakian (2018). In this approach, level of ambiguity is calculated by the volatility of probabilities. In authors' own words, *"…ambiguity can be measured by the volatility of probabilities, just as the degree of risk can be measured by the volatility of returns…"*.

Let the following triple set (Ω, δ, P) define our probability space for which Ω is the return state space, δ represents the σ-algebra associated with the subsets of Ω and P is a probability on δ ($P \in \mathcal{P}$ where $\mathcal{P}$ denotes the set of all probability measures). In a nested and similar manner, we

[12]Other recent studies that rely on divergence of market experts' forecasts in measuring Knightian uncertainty: Ulrich (2013) while studying inflation ambiguity and term premium in US government bond yields; Drechsler (2013) to quantify model uncertainty and to explain representative investor's portfolio formation decisions.

can simply define another triple set $(\mathcal{P}, \flat, \xi)$, elements of which respectively stand for state space of probabilities, measurable subsets within this new state space $\mathcal{P}$, and a probability measure on the new algebra.

In our study, the unknown spot index return is defined to be $r : \Omega \to \mathbb{R}$ with its density function $\varphi(r)$. In line with above notation, cumulative return probability is P ($P \in \mathcal{P}$). Expected return and associated variance can be calculated with expected probabilities as in Eq. (3).

$$E[r] \equiv \int E[\varphi(r)]rdr \quad \text{and} \quad Var[r] \equiv \int E[\varphi(r)](r - E[r])^2 dr \tag{3}$$

Following Brenner and Izhakian (2018), with the second-order (subjective) probability measure ξ which is grounded on credence, we define expected marginal and cumulative probabilities on this return respectively via Eq. (4) as

$$E[\varphi(r)] \equiv \int_{\mathcal{P}} \varphi(r)d\xi \quad \text{and} \quad E[P(r)] \equiv \int_{\mathcal{P}} P(r)d\xi \tag{4}$$

The intuition for the variance of the marginal probability is that the returns are not identically distributed over time (not independently as well) and the expectation of distributions are mainly driven by the variation in the set of priors as opposed to Bayesian approach where there is only a single prior. Put differently, probability of a return level falling in a certain interval varies depending on the priors over time. Variation in these probabilities can be formally represented by the variance of marginal probability defined in Eq. (5) and associated ambiguity measure is formulated as in Eq. (6). At a point in time, these priors are the previous return distributions constructed over a certain period; say a day. For instance, think of an empirical probability distribution derived via intraday returns with a specified measurement interval. If the each trading day has exactly the same distribution over a month, the investor will have only one prior and the ambiguity will be zero. As the distributions become less identical, set of priors (number of different probability distributions) will increase and that will make the outlook more ambiguous. As in Brenner and Izhakian (2018), we assume normally distributed daily returns and take the differing moments of the distributions as reference while calculating the ambiguity. Hence, level of ambiguity can be formulated as in Eq. (7) for the continuous case.

$$\mathrm{Var}[\varphi(r)] \equiv \int_{\mathcal{P}} (\varphi(r) - E[\varphi(r)])^2 d\xi \tag{5}$$

$$\mho^2[\mathrm{r}] = \int \mathrm{E}[\varphi(\mathrm{r})]\,\mathrm{Var}[\varphi(\mathrm{r})]\mathrm{dr}. \tag{6}$$

$$\mho^2[\mathrm{r}] = \int \mathrm{E}[\phi(r;\mu,\sigma)]\,\mathrm{Var}[\phi(r;\mu,\sigma)]dr \tag{7}$$

where $\phi(.)$ corresponds to normal probability density function.

To derive the ambiguity numbers from S&P500 index data, we create histograms for each individual day as suggested in our methodological guide. A typical open-close trading period is divided into bins with 5-minute intervals and respective returns are calculated. To construct the normally distributed daily priors (every P for each day), we refer to the mean (μ) and variance (σ^2) parameters from the 5-minute return distributions of each single day. These daily parameters are the most pivotal components of ambiguity measurement. The cost of shrinking the interval length further is the market micro-structure noise that contaminates the data and the staleness in spot index because of the non-synchronicity originating from the lead-lag effect in index constituents.

The phases in quantifying the ambiguity are explained in order. We first construct histograms at each separate day for return distributions. Second, we determine a daily return range (±6% in the present study) and divide it into 60 intervals (implying 0.02% return for each interval) as done in Brenner and Izhakian (2018). As a caveat for applications in markets where the extremes are experienced more frequently with larger intraday fluctuations (where the tails are thicker), expanding the base return range (e.g. ±10%) may be methodologically more appropriate to assign individual bins to higher levels of absolute returns. In the next step, probability of the returns coinciding with each bin is computed on every single day.

Technically, let $b_{i,r_{j,j+1}}$ represent the probability on a particular day i and for a certain bin bounded by the return cutoffs r_j and r_{j+1} where $i = 1, 2, \ldots, 22$ and $j = 0, 1, \ldots, 59$. For each day i, probability distribution (the daily prior) will have distinctive parameters μ_i and σ_i^2. As a matter of course, we will have a series of mean and standard deviation ratios $(\frac{\mu_1}{\sigma_1}, \frac{\mu_2}{\sigma_2}, \ldots, \frac{\mu_{22}}{\sigma_{22}})$ and we hold the assumption of t-distribution for these daily ratios; that is $\frac{\mu}{\sigma} \sim t(\varkappa, \daleth)$. As stressed in Brenner and Izhakian (2018), this assumption is central to computing expected probabilities and their pertinent variances. For the bins over the whole period, we calculate separate probabilities $b_{1,r_{jj+1}}, b_{2,r_{jj+1}}, \ldots, b_{22,r_{jj+1}}$ integrated over daily densities with corresponding $(\mu_1, \sigma_1^2), (\mu_2, \sigma_2^2), \ldots, (\mu_{22}, \sigma_{22}^2)$ parameters. We also calculate probabilities for the outer sections of return range boundary; b_{i,r_0} for the probability below -6% cutoff and $b_{i,1-r_{60}}$ for probability above 6% cutoff. Out of these computations, we calculate the mean and variance of probabilities at each single bin corresponding to the period; notationally $(\mu_{b_{j,j+1}}, \sigma^2_{b_{j,j+1}})$ for the bin with cutoffs r_j and r_{j+1}.

Ambiguity within a certain window length (which is 22 days in the present study) is estimated through Eq. (8); the discrete version of Eq. (7). Therein, r_0 and r_{60} respectively stand for -6% and 6% return boundaries and w in the scaling factor $\frac{1}{w(1-w)}$ is simply the bin size.

$$\begin{aligned}\mho^2[r] = \frac{1}{w(1-w)} \times (&E\left[\Phi\left(r_0;\mu,\sigma\right]\operatorname{Var}\left[\Phi\left(r_0;\mu,\sigma\right]\right.\right.\\ &+\sum_{i=1}^{60} E\left[\Phi\left(r_i;\mu,\sigma\right)-\Phi\left(r_{i-1};\mu,\sigma\right)\right]\operatorname{Var}\left[\Phi\left(r_i;\mu,\sigma\right)\right.\\ &\left.-\Phi\left(r_{i-1};\mu,\sigma\right)\right]+E\left[1-\Phi\left(r_{60};\mu,\sigma\right)\right]\operatorname{Var}\left[1-\Phi\left(r_{60};\mu,\sigma\right)\right]),\end{aligned} \tag{8}$$

Before the ambiguity expectation, we first calculate the realized ambiguities within a certain window size by moving one day forward at each calculation. Observe that these are not the realized daily ambiguities, they are rather the realized ambiguities for each window period and attached to the last days of each window. This way, we construct a time series of realized ambiguities with length $N - l + 1$ where N is the total number of days in our data span and l is the window size. With this time series of realized ambiguities at hand, we check the auto-correlations. In autoregressive (AR) univariate time series modelling, predictive information comes only from information embedded in the previous values of the variable. This modelling can be enriched by adding the lags of error term (ARMA). Motivated by the auto-correlations in realized ambiguity, we expect next day's ambiguity via Eq. (9) and Eq. (10). We decide on the lag selection by looking at the AICC (corrected AIC) values for different combinations of p and q with max lags of 10 for each ($p \times q = 100$ combinations).

$$\ln \mho_t = \psi_0 + \epsilon_t + \sum_{i=1}^{p} \psi_i \cdot \ln \mho_{t-1} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{t-1}. \tag{9}$$

$$\left(\mho^2_{t+1}\right)^E = E_t\left[\mho^2_{t+1}\right] = \exp\left(2\widehat{\ln \mho}_t + 2\operatorname{Var}\left[u_t\right]\right), \tag{10}$$

2.2.2. Model Estimation

As our base case, we use quantile regression (QR) approach to analyze the relationship between our variables of interest. This non-parametric technique is more appropriate especially when the explanatory variables are affecting different percentiles of the response variable distinctively. This is crucial to check the impact of ambiguity (and other independent variables as well) on different quantiles of the mispricing. Since there is a more complex relationship (rather than linear with constant variation) between the response and explanatory variables, QR is a better model for our purposes. In a similar fashion to ordinary least squares approach (OLS) in which we want to know the conditional mean of the dependent variable, we are interested in conditional quantiles of the dependent variable where the estimation mechanism rests on mean-absolute-deviation as opposed to mean-squared-errors. Hence, in the QR approach, parameter estimates are obtained by minimizing the sum of absolute deviations based on the quantiles denoted as τ.

Notation-wise, $Quantile_{\tau}(y|x) = X\beta_{\tau}$ is the formal description of quantile regression just as $E(y|x) = X\beta_{\mu}$ targets the conditional mean of the dependent variable. Cutoff points for the quantiles typically work on the ordered sequence of a variable. In the conditional set-up, these cutoff points change depending on the level of explanatory variables; one of which is the lagged market ambiguity in our analysis. With no distributional assumptions and being more robust to the presence of outliers, QR provides us with different coefficient estimates for different quantiles (as opposed to OLS) and help us understand varying association for different levels of response variable.

For both absolute mispricing and boundary violations, we use the following model set-up in our analysis.

$$|MP_t| = \beta_0 + \beta_1 Amb^E_{t-1} + \beta_2 VIX_{t-1} + \beta_3 DR_t + \beta_4 VOL_t + \beta_5 VVOL_t + \varepsilon_t \tag{11}$$

where Amb^E_t is the expected ambiguity for the next day ($\mho^2_{t+1}{}^E$), DR_t is the remaining days to contract expiry, VOL_t is spot volatility and $VVOL_t$ is the volatility of volatility. We analyze positive and negative mispricings separately as the degree of ambiguity can have differing affects on futures mispricing. It is widely documented that investors act more aggressively in bad market conditions. This asymmetric reaction rests on agents' psychological inclination to adapt themselves to worst-case scenarios.[13] We run our model with 0.10 increments up to 0.80 and with 0.05 increments thereafter: $\tau = [0.10, 0.20, ..., 0.80, 0.85, 0.90, 0.95]$. We model boundary violations (*Bvio*) similarly in QR set-up. For positive and negative mispricing, dependent variable naturally contains too many zeros which make the distribution highly skewed and unsuitable for OLS analysis. It is a $semi-continuous$ series with $true\ zeros$ and we cannot handle this feature of the data via transformation or discretization as they will offer limited cure with considerable loss of information.[14]

$$|Bvio_t| = \beta_0 + \beta_1 Amb^E_{t-1} + \beta_2 VIX_{t-1} + \beta_3 DR_t + \beta_4 VOL_t + \beta_5 VVOL_t + \varepsilon_t \quad (12)$$

After placing the rationale of QR approach, it is equally crucial to state that our analysis covers a very large period which naturally raises the question of parameter instability. To grasp the evolution of the association between the level of mispricing and the market uncertainty, we also check the emergence of parameter significance and direction for market ambiguity. To do that, we apply rolling window QR regressions over the entire period.

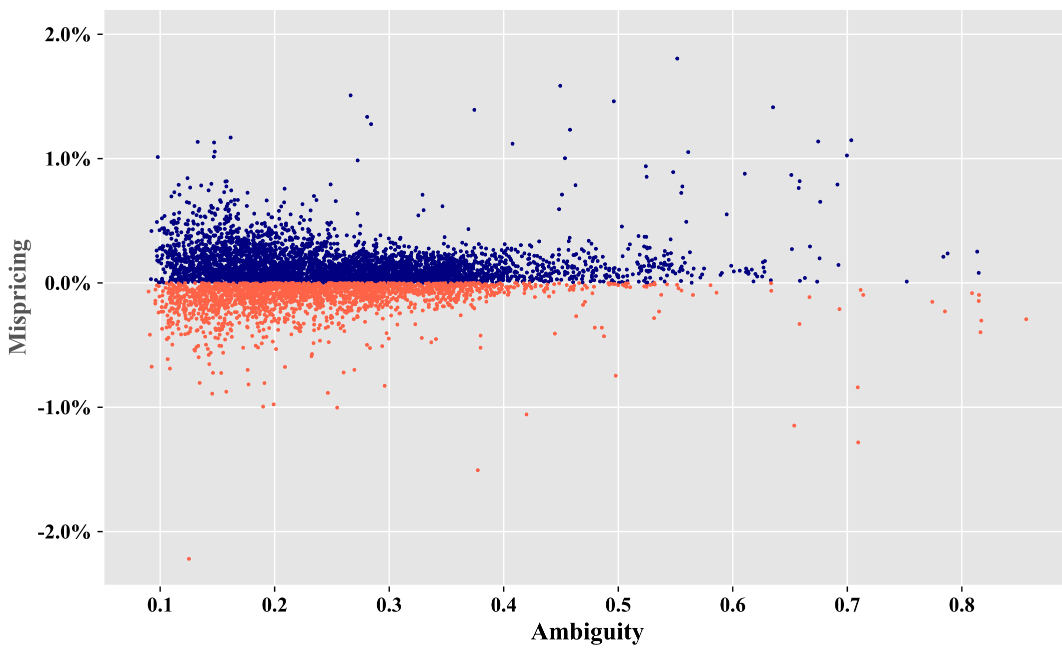
3. Empirical Findings

Our empirical analyses divulge striking findings on the asymmetric and time-varying effects of ambiguity in financial markets. In the first place, we separately run our regressions for the whole period for all absolute mispricings as well as mispricings which violate no-arbitrage boundaries. Main findings in Table 1 reveal distinctive mispricing reactions for different ambiguity regimes prevalent in the market. It tabulates results for all absolute mispricings and we show that there is a clear positive relationship between the deviations from the theoretical index futures prices and the surrounding market ambiguity. Results also show that lagged ambiguity is significant almost for all quantiles and the coefficient generally gets larger as we move towards the highest quantile. Reported findings in Table 1 also demonstrate that spot volatility, implied volatility, days

[13] Among others, see for instance Epstein and Schneider (2008) and Gollier (2011) for further discussion on ambiguity and investor reactions.

[14] Data discretization means converting the data to categorical variable by creating partitions and assinging ordered values to all non-zero observations. Then, required analysis can be completed via (ordered) logistic regression model. Tobit regression approach is also not compatible since it also assumes normality for the residuals and zeros in our *Bvio* variable are not for the concealed observations.

Figure 1: Positive and Negative Mispricings vs Ambiguity



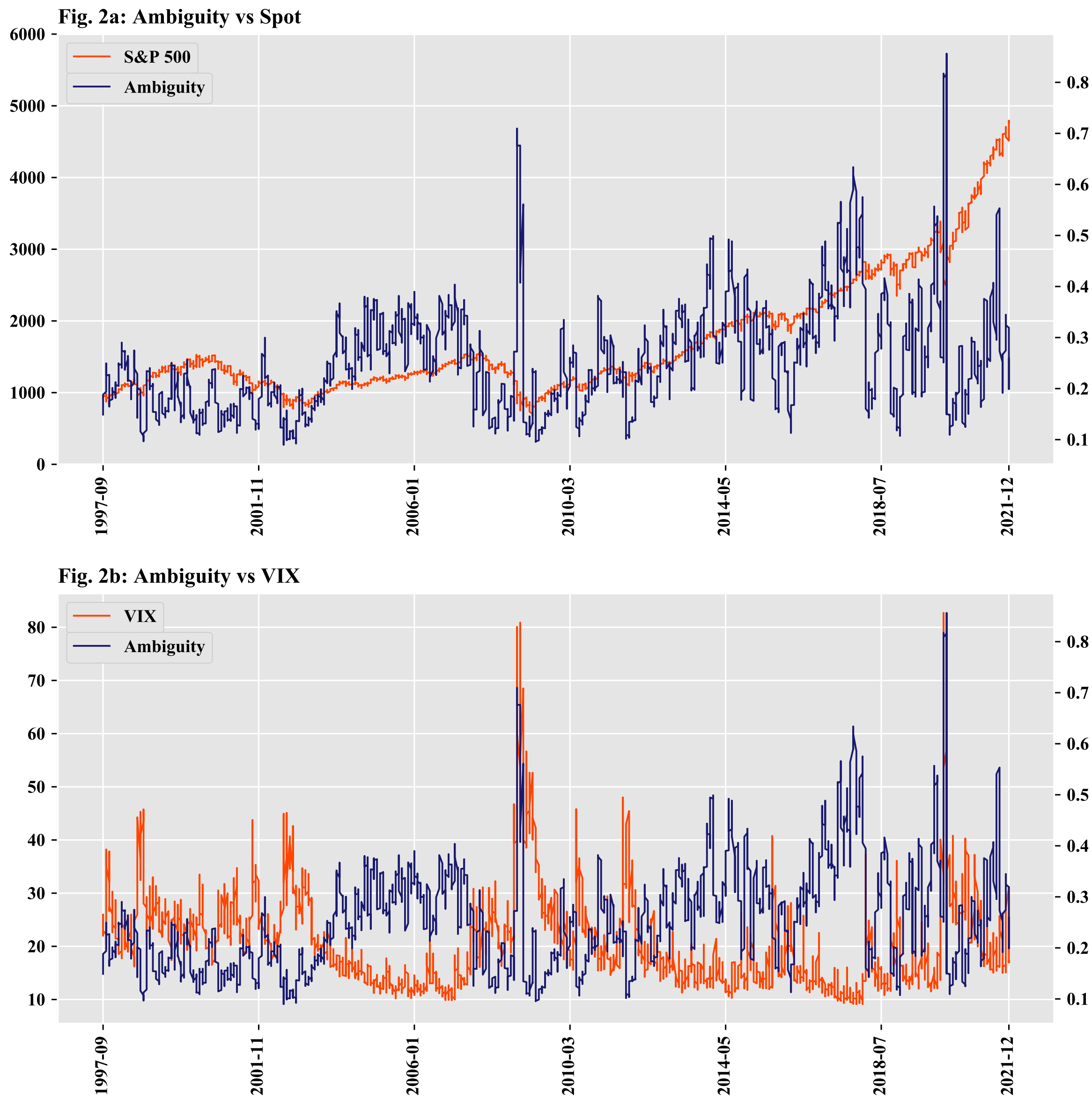
Notes: ....

to contract maturity and volatility of the spot volatility have explanatory powers in many quantiles of the mispricing in line with the relevant literature.

Table 2 reports the same results for boundary violations. The reason for the first meaningful quantiles in Table 1 and Table 2 not being aligned (τ = 0.10 vs τ = 0.70) is the number of zeros in latter data set; with boundary violations we only focus on mispricing magnitudes which are above the arbitrage cost level.

On the other hand, it is of more interest to see if the market ambiguity has differing explanatory implications in different periods of the whole data span. Renowned as parameter instability, this is especially important when data at hand cover decades in which the varying association is highly possible. To expose the concealed time-variation in market ambiguity coefficients, we run rolling window Q-regressions and discover a very crucial trend in the relationship between ambiguity and absolute mispricing.

Figure 2: Ambiguity, Spot Index and VIX



3.1. Varying Ambiguity Effect

Results in Panel A of Table 1 show that absolute mispricing has a positive relationship with market ambiguity. Repeating the same analysis for positive and negative mispricing even evinces that

Table 1

QR Results for Index Futures Mispricing

Notes: Table reports QR results derived from the following model: $|MP_t| = \beta_0 + \beta_1 Amb^E_{t-1} + \beta_2 VIX_{t-1} + \beta_3 DR_t + \beta_4 VOL_t + \beta_5 VVOL_t + \varepsilon_t$. t-statistics are reported in parenthesis. VIX and Days Remaining are scaled by 1/1,000 and 1/1,000,000 respectively. We use the lag of AMB_t in all regressioins as it is the expectation for the next day's ambiguity.

ABSOLUTE MISPRICING &AMBIGUITY											
Quantiles	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.85**	**0.9**	**0.95**
$Intercept$	-0.0001	-0.0002	-0.0003	-0.0005	-0.0005	-0.0006	-0.0008	-0.001	-0.0011	-0.0011	-0.0014
	(-2.79)	(-4.73)	(-5.28)	(-7.5)	(-6.14)	(-6.8)	(-8.42)	(-8.57)	(-7.72)	(-6.74)	(-6.38)
AMB_{t-1}	0.0001	0.0001	0.0001	0.0004	0.0002	0.0003	0.0006	0.0007	0.0007	0.0004	0.001
	(0.77)	(0.91)	(1.25)	(3.1)	(1.26)	(1.96)	(3.29)	(2.96)	(2.49)	(1.15)	(2.25)
VIX_{t-1}	0.0015	0.0072	0.0110	0.0218	0.0352	0.0448	0.0497	0.0642	0.0743	0.0797	0.0915
	(0.8)	(3.1)	(3.89)	(6.7)	(9.16)	(10.44)	(10.51)	(11.75)	(11.03)	(10.73)	(9.07)
DR_t	2.378	4.930	6.609	8.004	9.215	10.170	11.907	13.344	14.392	16.308	17.639
	(9.32)	(15.17)	(16.84)	(17.24)	(16.6)	(16.09)	(16.74)	(15.81)	(13.55)	(13.81)	(10.33)
VOL_t	0.0095	0.0119	0.0194	0.0216	0.0243	0.0403	0.0662	0.0804	0.0847	0.1044	0.1381
	(3.4)	(3.36)	(4.67)	(4.5)	(4.34)	(6.6)	(9.98)	(10.46)	(8.67)	(9.47)	(8.75)
$VVOL_t$	0.002	0.0021	0.0063	0.0066	-0.0185	-0.0342	-0.0163	0.0041	0.0416	0.0507	0.0994
	(0.27)	(0.23)	(0.6)	(0.53)	(-1.26)	(-2.08)	(-0.91)	(0.2)	(1.67)	(1.83)	(2.64)
$Pse.R\hat{2}$	0.02	0.03	0.05	0.06	0.08	0.10	0.13	0.17	0.20	0.24	0.30

Table 2

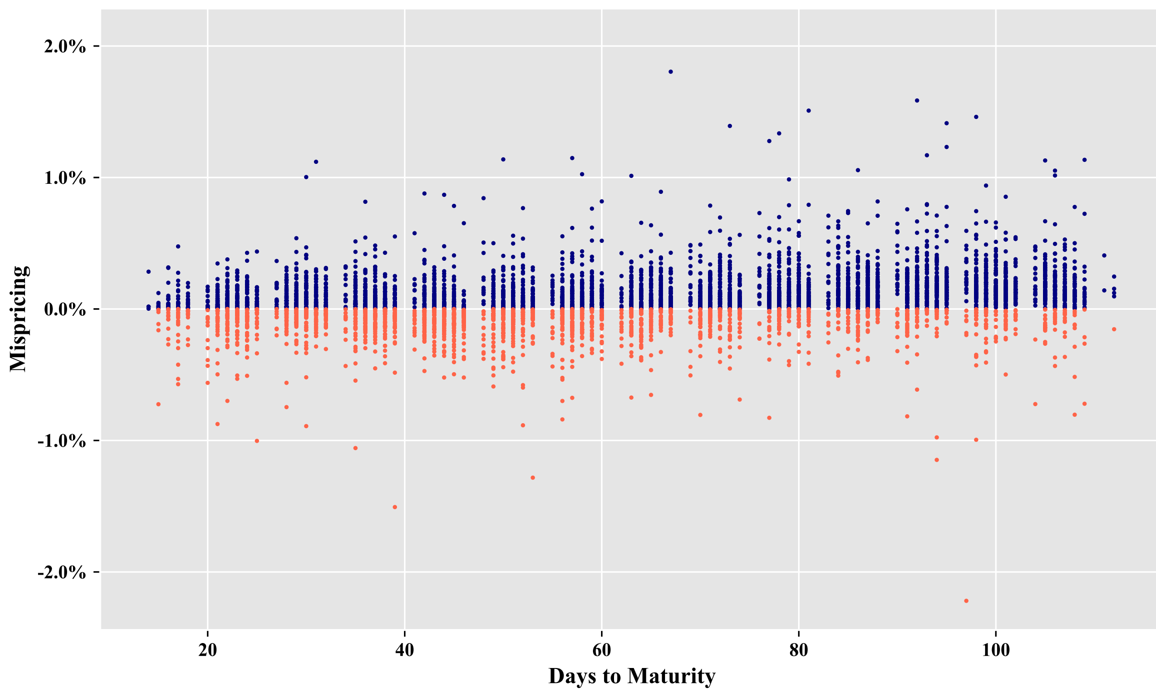
QR Results for Boundary Violating Mispricing

Notes: Table reports QR results derived from the following model: $|Bvio_t| = \beta_0 + \beta_1 Amb^E_{t-1} + \beta_2 VIX_{t-1} + \beta_3 DR_t + \beta_4 VOL_t + \beta_5 VVOL_t + \varepsilon_t$. t-statistics are reported in parenthesis. VIX and Days Remaining are scaled by 1/1,000 and 1/1,000,000 respectively. We use the lag of AMB_t in all regressioins as it is the expectation for the next day's ambiguity. We use 0.2% of spot index level as our cost threshold. All violations are in absolute terms.

ALL BOUNDARY VIOLATIONS & AMBIGUITY					
Quantiles	**0.7**	**0.8**	**0.85**	**0.9**	**0.95**
$Intercept$	-0.0019	-0.0023	-0.0023	-0.0019	-0.0022
	(-37.41)	(-17.99)	(-14.31)	(-10.24)	(-8.49)
AMB_{t-1}	0.0016	0.0005	0.0002	-0.0007	-0.0001
	(16)	(1.88)	(0.6)	(-1.76)	(-0.19)
VIX_{t-1}	0.0607	0.1077	0.1093	0.1039	0.1058
	(23.5)	(16.85)	(13.81)	(11.42)	(8.8)
DR_t	2.763	13.458	20.267	24.696	27.284
	(7.15)	(12.83)	(15.31)	(16.81)	(13.7)
VOL_t	0.0842	0.0795	0.0781	0.0885	0.1292
	(23.29)	(8.64)	(6.81)	(6.54)	(6.89)
$VVOL_t$	-0.0042	-0.0054	0.0523	0.0765	0.1641
	(-0.42)	(-0.22)	(1.75)	(2.24)	(3.63)
$Pse.R^2$	0.07	0.21	0.21	0.23	0.28

the relationship is different for negative and positive mispricing; it turns to negative for absolute negative mispricings in the index futures. Our extensive analysis on sufficiently large different

Figure 3: Positive and Negative Mispricing vs Remaining Contract Life



subperiods further revealed different relationships within positive mispricing as well as negative mispricing analysis. Results of these analyses indicate the parameter instability in our explanatory variables of interest. To comprehend the evolution of this varying relationship precisely, we apply local quantile regressions with a rolling window set-up with 60 months of window length and 3 months of rolling step size. Selected window length enables us to conduct our analysis with adequate number of data. Step size is actually compatible with the duration of trading in front-month index futures contracts. As can be seen in Figure 4, there is a clear upward trend for the ambiguity t-statistics with the sign of ambiguity coefficients gradually turning to positive from negative over years. We confirm this trend in our unreported findings if we further shorten the window length down to 24 months or other interim lengths. With shorter window sizes, t-stats naturally fluctuate more but the upward trend is preserved.

Months on the x-axis of Figure 4 are the last months of QR window. Ambiguity coefficient is negative at the beginning but there is a sharp increase around 2005. After fluctuating near zero roughly between 2005 and 2008, this association turns to positive with the financial crises in 2008. T-statistics values also have a clear upward trend over the years for all quantiles (see Figure 4). It is quite obvious that upward movement in t-stat values is re-sparked after August 2008 as the acute times of 2008 financial crisis enter our window of regressions. When the new periods of financial crises get appended to our window, ambiguity coefficients first become positive and climax once the regression window contains only the post-crises period. In spite of the sharp decline thereafter, we can still see the upward movement in statistical significance after roughly 2014 at all quantiles. On the whole, this time-varying anlaysis takes our study to an interesting plateau and incentivizes us for a deeper investigation for the driving market forces behind the switching sign of this association between ambiguity and mispricing.

As the ambiguity soars, agents incline towards safer assets and exert selling pressure on the market leading to abrupt changes in prices as the decisions are shaped according to worst-case scenarios. It is widely documented that spot market and index futures do not track each other perfectly and there is a powerful price discovery for index futures in the leading spot market movements.[15] Adjustment of spot index level to new market conditions is comparatively ponderous due to large number of index constituents and some market imperfections like short-sale restrictions. In contrast to staleness in spot index levels, futures prices are more agile and volatile since it is only a single asset with advantageous cost structure and free of short-sale restrictions. It is thus quite straightforward that mispricing in index futures is highly dependent on futures market prices in the first place. Hence, either in tumbling or in bull markets, absolute mispricing levels will be governed primarily by the movements in futures markets. After all, mispricing means maladaptation between the spot-led theoretical futures price and the market futures price itself. A positive relationship between ambiguity and mispricing implies that those two values fall more apart from each other as the ambiguity climbs whereas a negative relationship implies that these values converge to each other amid soaring market ambiguity. Our time-varying analysis reveals that ambiguity has a varying predictive role for index futures mispricing with negative coefficients rapidly converging to zero boundary after roughly 2005 and becoming positive after broadly the financial crises. Based on the idea that mispricing is chiefly stirred by the participants of futures markets, it will be nice to keep futures markets under magnifying glass.

[15]Refer for instance to Brenner et al. (1989) and Hasbrouck (2003) for discussions on spot market and index futures pricing dynamics.

It is evident that investors are using futures markets for hedging purposes against unfavorable price movements in the underlying asset or indicator. As the activity in futures market increases as a result of rising concerns about potential losses, market reactions become sharper may it be induced by rising volatility levels or the uncertainty. In other words, enhanced futures markets activity with vigilant investors increases the likelihood of theoretical and market futures prices deviating more from each other. Along with these, we find rising ambiguity levels during upward market movements as depicted in the upper panel of Figure 2 and this is compatible with the results of Brenner and Izhakian (2018) as they similarly report positive association between price run-ups in the market and higher ambiguity levels. Common wisdom associates higher ambiguity with market downturns. Similar to Brenner and Izhakian (2018) however, we conjecture that sustained upward trend perturbs investors and intensifies the concern of a looming market downturn as they suspect the surge in the market is due to noise and it is not backed by the improvements in stock fundamentals.

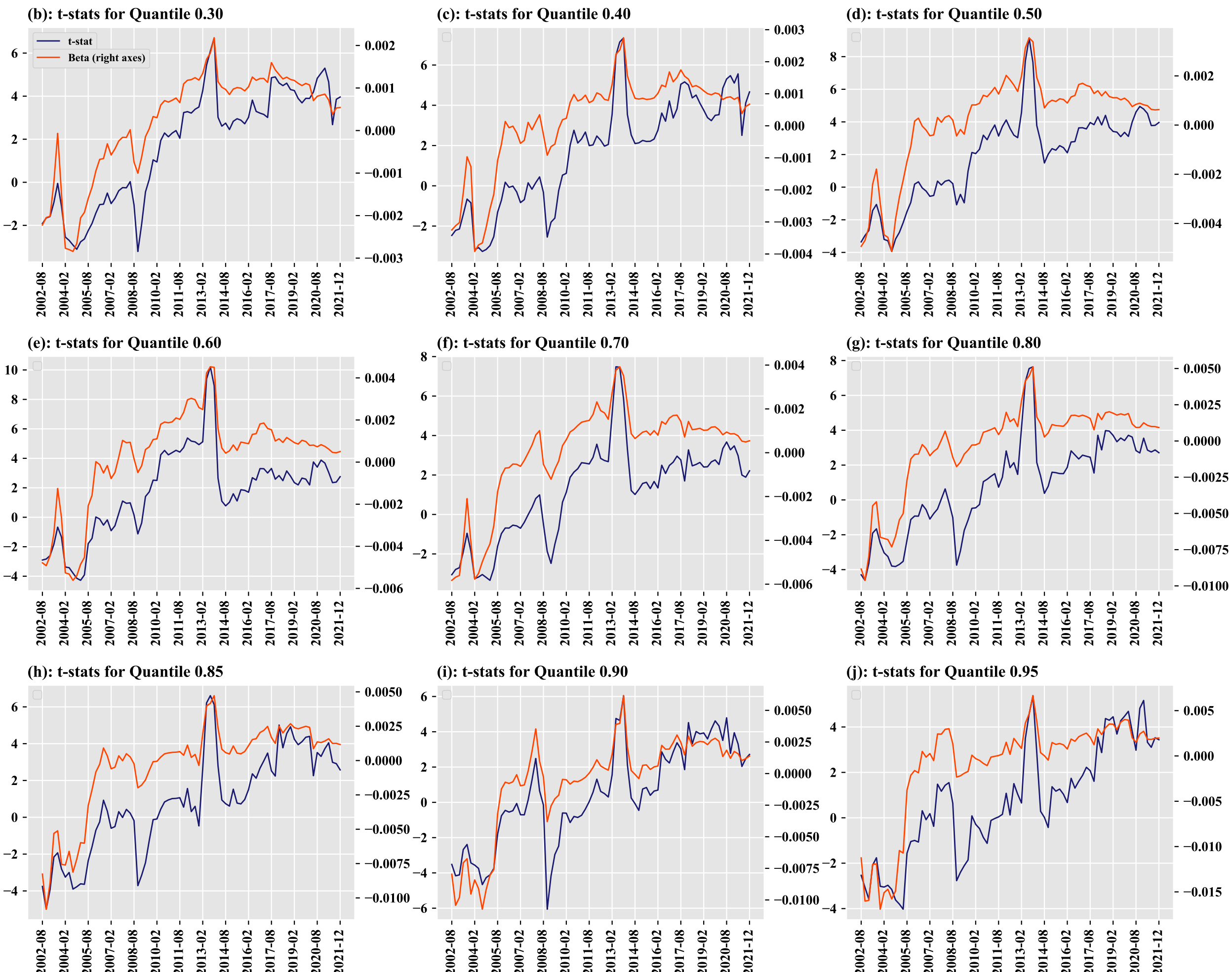
To gauge the degree of activity in futures market, we can look at open interests in futures contracts as an appropriate proxy.

4. Conclusion

This paper tries to improve our understanding about the mispricing in S&P 500 index futures contracts under market ambiguity. We report that mispricing in E-mini futures contracts have time-varying association with prevalent market uncertainty in which the the trend in this association switches sign from negative to positive over the past three decades. With a quantile regression approach, we are also able to see the degree of this relationship for different levels of mispricing and market ambiguity.

PS: These are our first findings and we are still working on how market ambiguity permeates the market. One of the issues we are trying to tackle is the price pressures created by hedgers' demands when ambiguity soars; especially the demand of index option market makers and leveraged ETFs to hedge short gamma exposures.

Figure 4:
Direction of Ambiguity Effect



Notes: This graph shows time-varying coefficients and t-statistics from rolling window QR outputs for different quantiles with a Window length of 60 months and rolling step size of 3 months. Data cover the period September 1997 - December 2021. Months on the x-axis are the last months of the window.

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